How to draw graphs in the Fourier series?

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How to draw graphs in the Fourier series?

Here I'll show how to draw the graphs in the Fourier series.So here you go!

Solved examples of how to draw graphs in the Fourier series

Disclaimer: None of these examples is mine. I have chosen these from some books. I have also given the due reference at the end of the post.

So here is the first example of how to draw graphs in the Fourier series.

Example 1

According to [Stroud and Booth (2011)]*, "

$$\begin{aligned} f(x) &= x^2 \quad 0 \leq x < 2 \\ &= (6-x) \quad 2 \leq x < 10 \\ f(x+10) &= f(x) \quad \text{for } -20 \leq x \leq 20. \end{aligned}$$

"

Solution

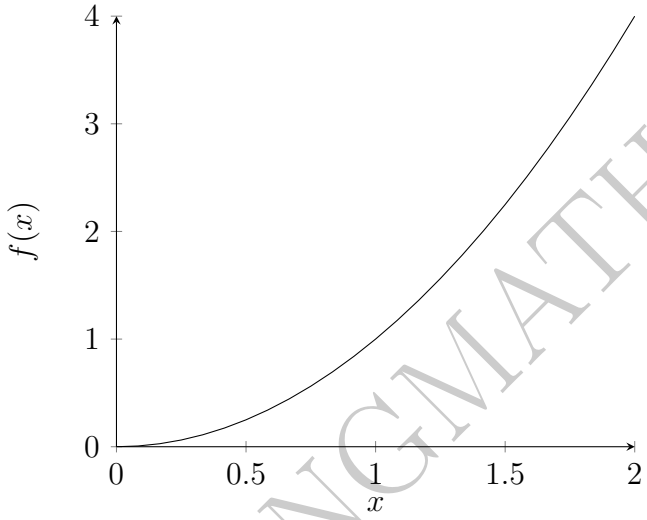
Now here the given function is

$$\begin{aligned} f(x) &= x^2 \quad 0 \le x < 2 \\ &= (6-x) \quad 2 \le x < 10 \\ f(x+10) &= f(x) \quad \text{for } -20 \le x \le 20. \end{aligned}$$

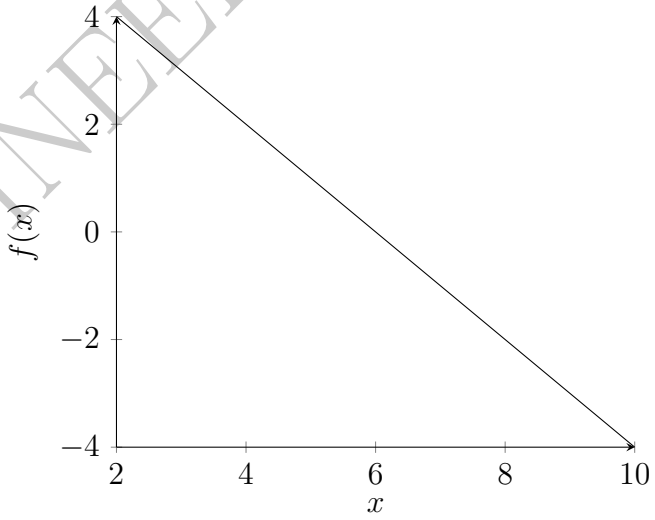
As I can see, the function $f(x)$ has two different values. One is x^2 and the other one is $(6-x)$. When the value of x is between 0 and 2, the value of $f(x)$ is x^2. So, first of all, I'll draw the graph of $f(x) = x^2$ when x is in between 0 and 2.

Step 1

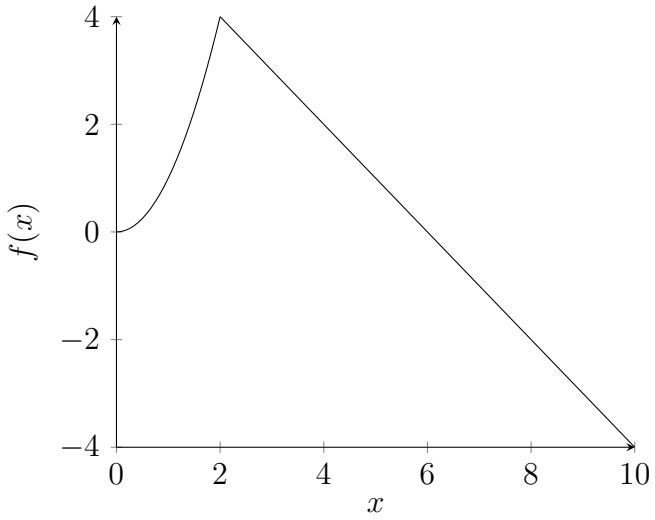
Thus the graph of $f(x)$, when x is in between 0 and 2, looks like the following:



Now I can see, when x is between 2 and 10, the value of $f(x)$ is $(6-x)$.So, I'll draw the graph of $f(x) = 6 - x$ when x is in between 2 and 10.



Now I'll combine these two graphs. So this gives



Also I can see that $f(x+10) = f(x)$ for $-20 \leq x \leq 20$. So this means the function $f(x)$ has a period of 10.

Now I'll draw the graph of $f(x)$ when x is in between 10 and 20.

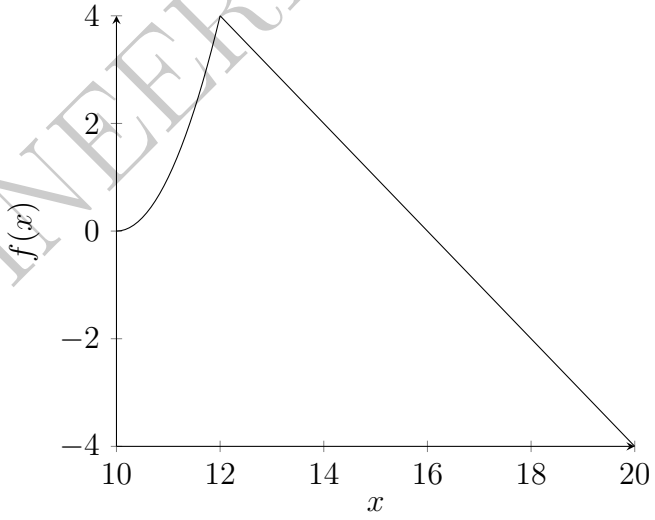
Step 2

Since the function $f(x)$ has a period 10, the function $f(x)$ will repeat itself in the interval $[10, 20)$.

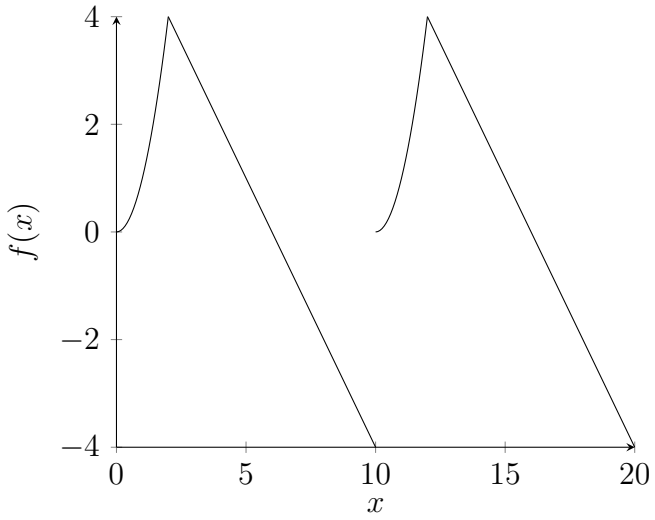
So the function $f(x)$ will be

$$\begin{aligned} f(x) &= x^2 \quad 10 \leq x < 12 \\ &= (6-x) \quad 12 \leq x < 20. \end{aligned}$$

Next, I'll draw the graph of $f(x)$ in this interval.



Therefore I can say that the graph of $f(x)$, when x is in between 0 and 20, looks like



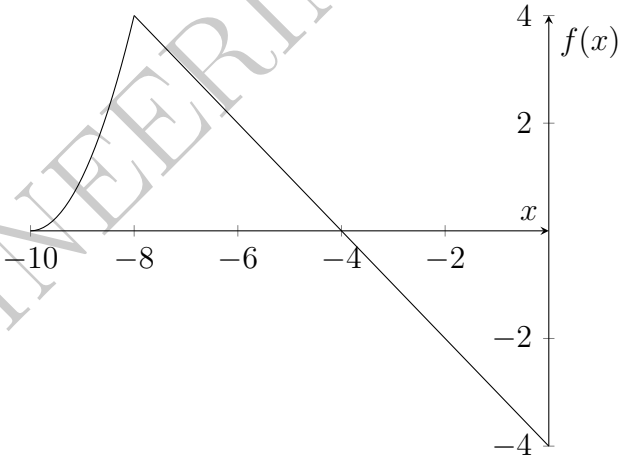
Next, I'll draw the graph of $f(x)$ when x is in between 0 and -20. At first, I'll draw the graph of $f(x)$ when x is in between 0 and -10. Then I'll draw the graph of $f(x)$ when x is in between -10 and -20.

Step 3

Since the function $f(x)$ has a period 10, $f(x)$ will repeat itself in the interval $[-10, 0)$. So the function $f(x)$ will be

$$\begin{aligned} f(x) &= x^2 \quad -10 \le x < -8 \\ &= 6 - x \quad -8 \le x < 0. \end{aligned}$$

Next, I'll draw the graph of $f(x)$ in this interval. So it will look like



Now I'll draw the graph of $f(x)$ when x is in between -20 and -10.

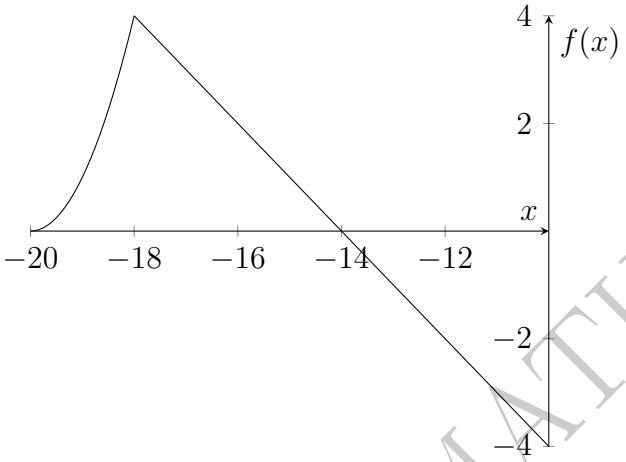
Step 4

As I have mentioned before, the function $f(x)$ has a period 10. So the function $f(x)$ will repeat itself in the interval $[-20, -10)$.

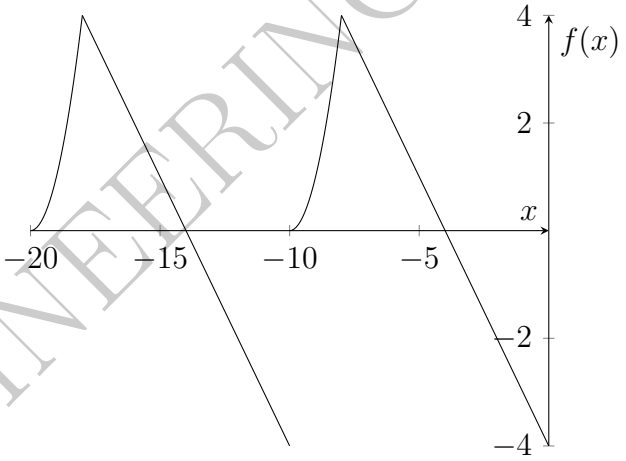
Thus the function $f(x)$ will be

$$\begin{aligned} f(x) &= x^2 \quad -20 \leq x < -18 \\ &= 6 - x \quad -18 \leq x < -10. \end{aligned}$$

Next, I'll draw the graph of $f(x)$ in this interval. So it will look like

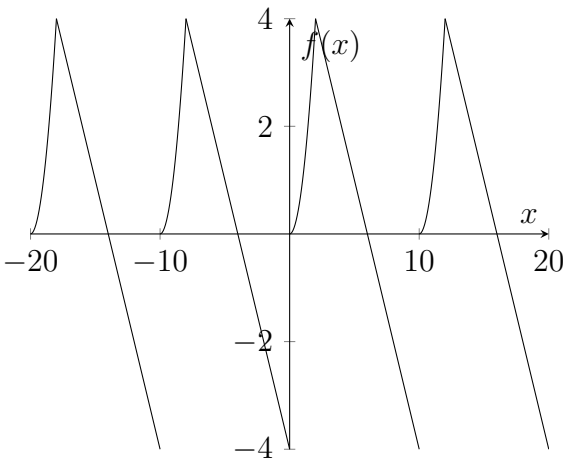


Therefore I can say that the graph of $f(x)$, when x is in between -20 and 0, looks like



Step 5

At the end, I'll bring together all these graphs. So the graph of $f(x)$ in the interval of -20 to 20 looks like as follows:



And this is the answer to this example.Now I'll give another example.

Example 2

According to [Stroud and Booth (2011)]*, "

$$\begin{aligned} f(x) &= x^3 \quad 0 \le x < 2 \\ &= 8 \quad 2 \le x < 3 \\ &= (5-x)^3 \quad 3 \le x < 5 \\ f(x+5) &= f(x) \quad \text{for } -10 \le x \le 10. \end{aligned}$$

"

Solution

Now here the given function is

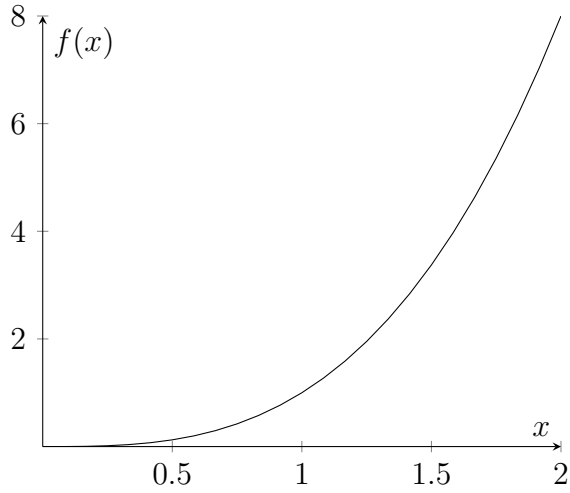
$$\begin{aligned} f(x) &= x^3 \quad 0 \le x < 2 \\ &= 8 \quad 2 \le x < 3 \\ &= (5-x)^3 \quad 3 \le x < 5 \\ f(x+5) &= f(x) \quad \text{for } -10 \le x \le 10. \end{aligned}$$

As I can see, the function $f(x)$ has three different values. One is x^3 and the other two are $8, (5-x)^3$. When the value of x is between 0 and 2, the value of $f(x)$ is x^3.

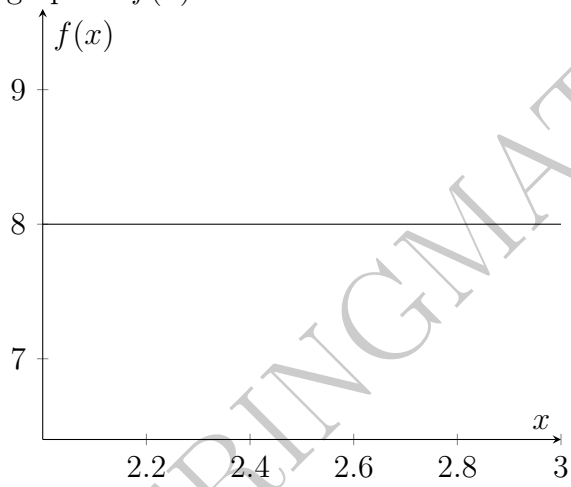
So, first of all, I'll draw the graph of $f(x) = x^3$ when x is in between 0 and 2.

Step 1

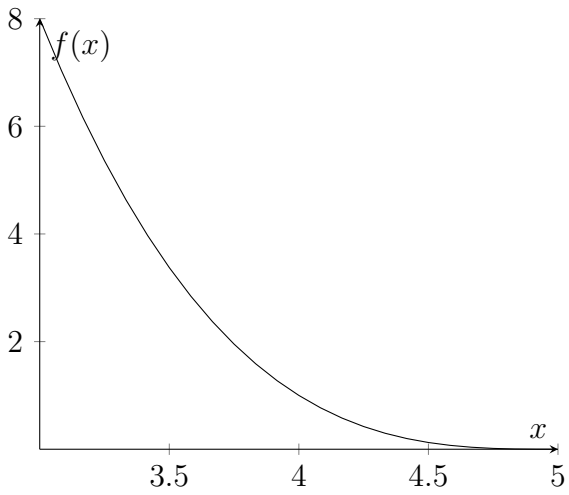
Thus the graph of $f(x)$, when x is in between 0 and 2, looks like the following:



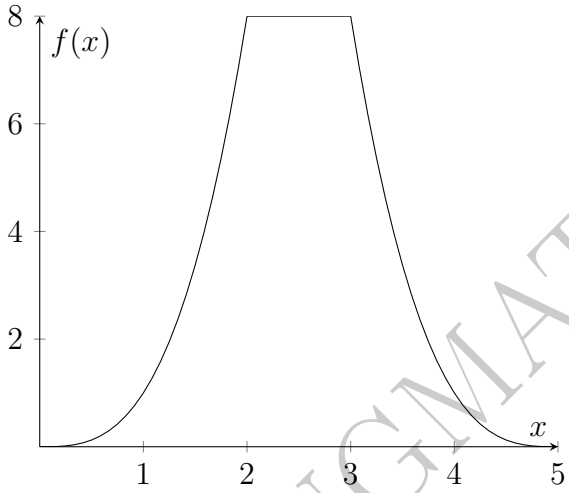
Now I can see, when x is between 2 and 3, the value of $f(x)$ is 8.So, I'll draw the graph of $f(x) = 8$ when x is in between 2 and 3. And this looks like



Also the value of $f(x)$ is $(5 - x)^3$, when x is in between 3 and 5. So the graph of $f(x) = (5 - x)^3$, when x is in between 3 and 5, looks like



Now I'll combine these three graphs. So this gives



Also I can see that $f(x+5) = f(x)$ for $-10 \leq x \leq 10$. So this means the function $f(x)$ has a period of 5.

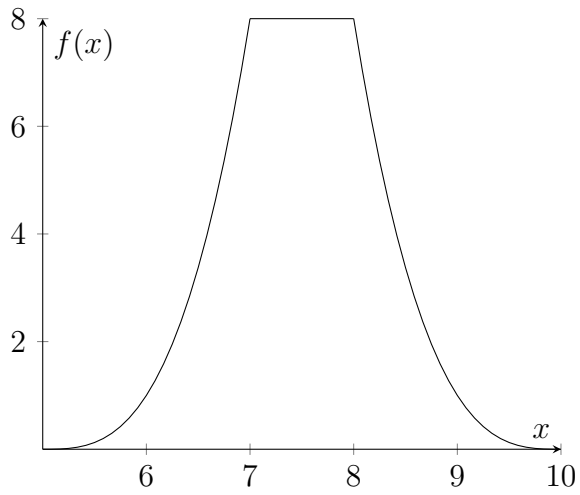
Now I'll draw the graph of $f(x)$ when x is in between 5 and 10.

Step 2

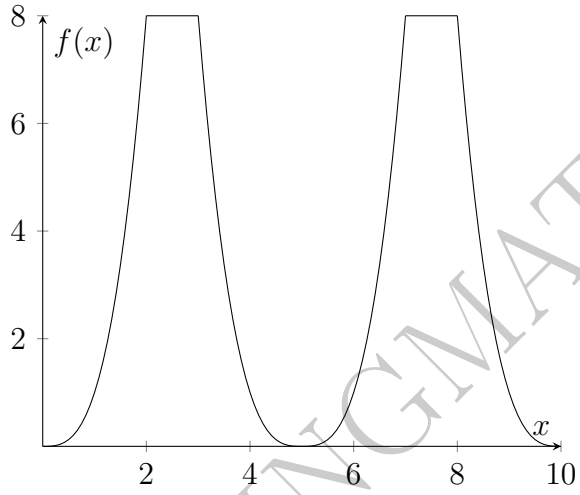
Since the function $f(x)$ has a period 5, the function $f(x)$ will repeat itself in the interval $[5, 10)$. So the function $f(x)$ will be

$$\begin{aligned} f(x) &= x^3 \quad 5 \leq x < 7 \\ &= 8 \quad 7 \leq x < 8 \\ &= (5-x)^3 \quad 8 \leq x < 10 \end{aligned}$$

Next, I'll draw the graph of $f(x)$ in this interval.



Therefore I can say that the graph of $f(x)$, when x is in between 0 and 10, looks like



Next, I'll draw the graph of $f(x)$ when x is in between 0 and -10. At first, I'll draw the graph of $f(x)$ when x is in between 0 and -5. Then I'll draw the graph of $f(x)$ when x is in between -5 and -10.

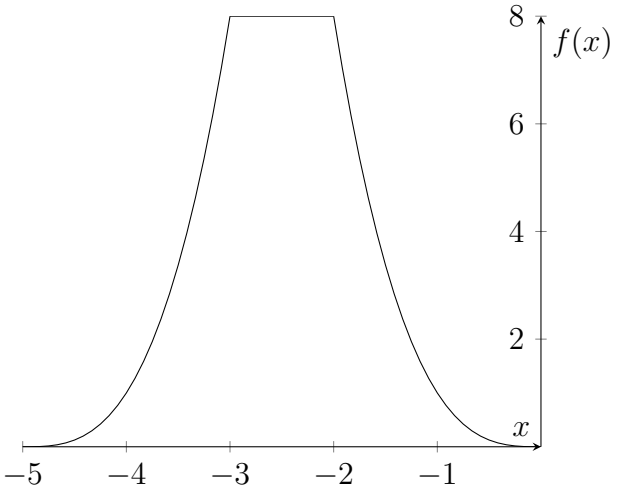
Step 3

Since the function $f(x)$ has a period 5, $f(x)$ will repeat itself in the interval $[-5, 0)$. So the function $f(x)$ will be

$$\begin{aligned} f(x) &= x^3 \quad -5 \le x < -3 \\ &= 8 \quad -3 \le x < -2 \\ &= (5-x)^3 \quad -2 \le x < 0 \end{aligned}$$

.

Next, I'll draw the graph of $f(x)$ in this interval. So it will look like



Now I'll draw the graph of $f(x)$ when x is in between -10 and -5.

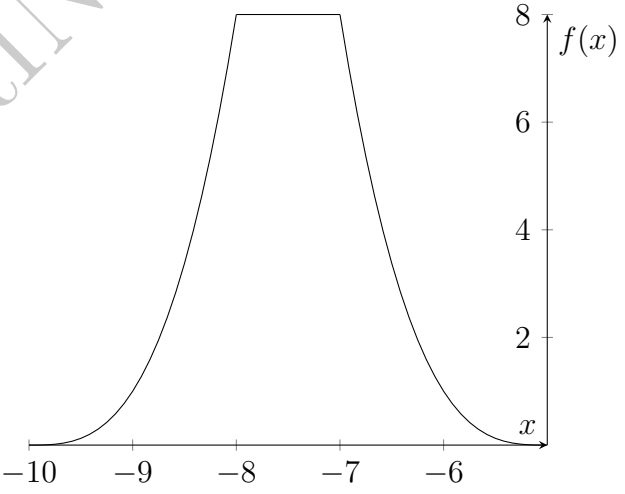
Step 4

As I have mentioned before, the function $f(x)$ has a period 5. So the function $f(x)$ will repeat itself in the interval $[-10, -5)$. Thus the function $f(x)$ will be

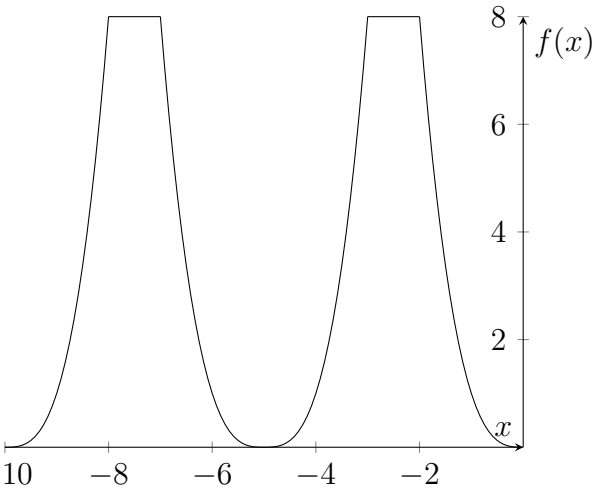
$$\begin{aligned} f(x) &= x^3 \quad -10 \le x < -8 \\ &= 8 \quad -8 \le x < -7 \\ &= (5-x)^3 \quad -7 \le x < -5. \end{aligned}$$

Next, I'll draw the graph of $f(x)$ in this interval.

So it will look like



Therefore I can say that the graph of $f(x)$, when x is in between -10 and 0,



looks like

And this is the answer to this example. Now I'll give another example.

Example 3

According to [Stroud and Booth (2011)], "Draw the graph of

$$\begin{aligned} f(x) &= (x+4)^2 \quad -4 \le x < -2 \\ &= 4-2x \quad -2 \le x < 0 \\ f(x+4) &= f(x) \text{ for } -8 \le x \le 8. \end{aligned}$$

"

Solution

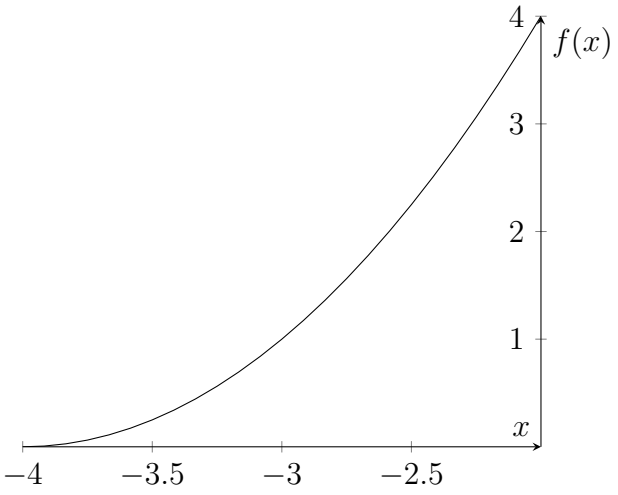
Now here the given function is

$$\begin{aligned} f(x) &= (x+4)^2 \quad -4 \le x < -2 \\ &= 4-2x \quad -2 \le x < 0 \\ f(x+4) &= f(x) \text{ for } -8 \le x \le 8. \end{aligned}$$

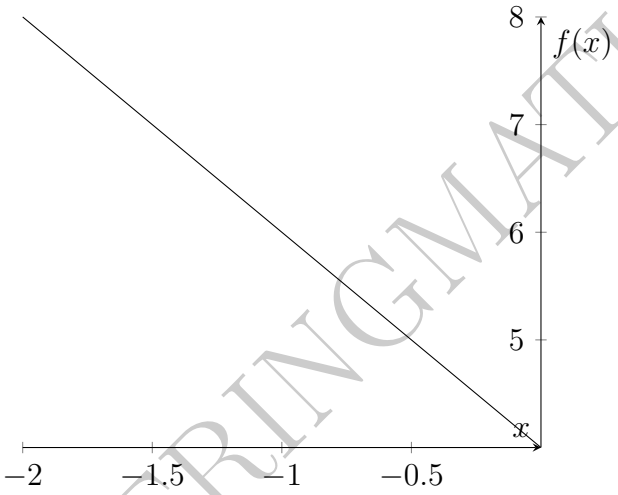
As I can see, the function $f(x)$ has two different values. One is $(x-4)^2$ and the other one is $(4-2x)$. When the value of x is between -4 and -2, the value of $f(x)$ is $(x+4)^2$. So, first of all, I'll draw the graph of $f(x) = (x+4)^2$ when x is in between -4 and -2.

Step 1

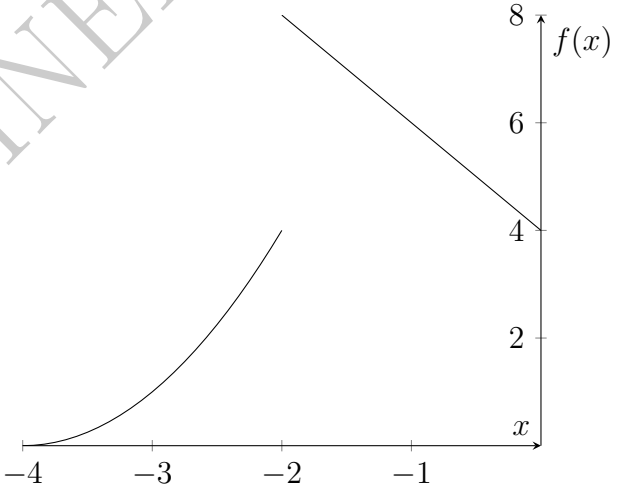
Thus the graph of $f(x)$, when x is in between -4 and -2, looks like the following:



Now I can see, when x is between -2 and 0, the value of $f(x)$ is $(4-2x)$. So, I'll draw the graph of $f(x) = 4 - 2x$ when x is in between -2 and 0.



Now I'll combine these two graphs. So this gives



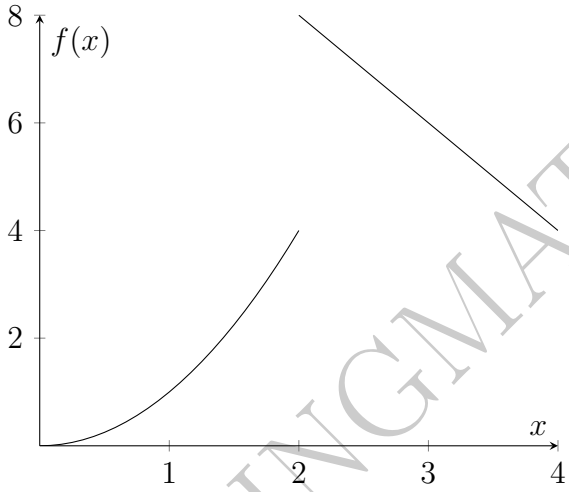
Also I can see that $f(x+4) = f(x)$ for $-8 \leq x \leq 8$. So this means the function $f(x)$ has a period of 4. Now I'll draw the graph of $f(x)$ when x is in between 0 and 4.

Step 2

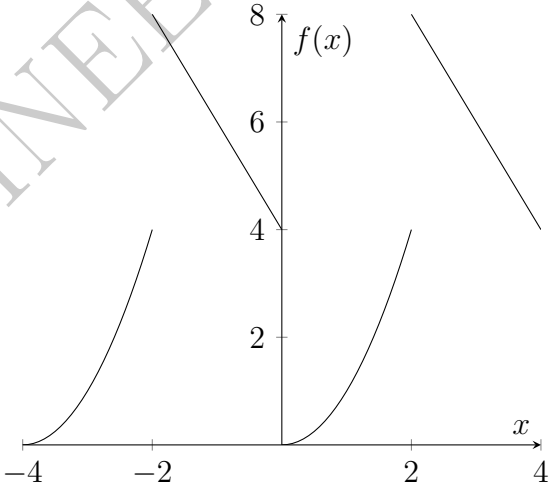
Since the function $f(x)$ has a period 4, the function $f(x)$ will repeat itself in the interval $[0, 4)$. So the function $f(x)$ will be

$$\begin{aligned} f(x) &= (x+4)^2 \quad 0 \leq x < 2 \\ &= 4 - 2x \quad 2 \leq x < 4. \end{aligned}$$

Next, I'll draw the graph of $f(x)$ in this interval.



Therefore I can say that the graph of $f(x)$, when x is in between -4 and 4, looks like



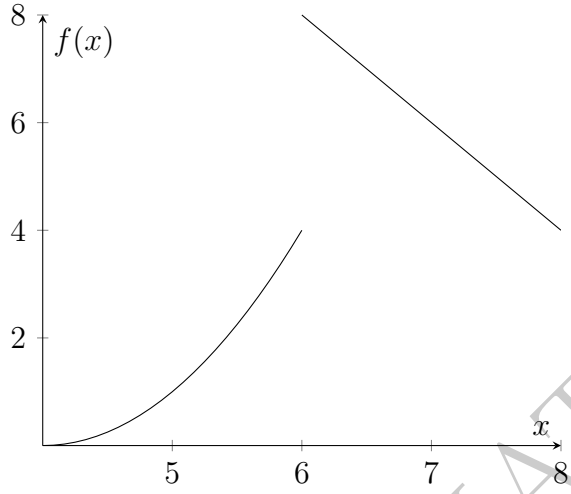
Next, I'll draw the graph of $f(x)$ when x is in between 4 and 8.

Step 3

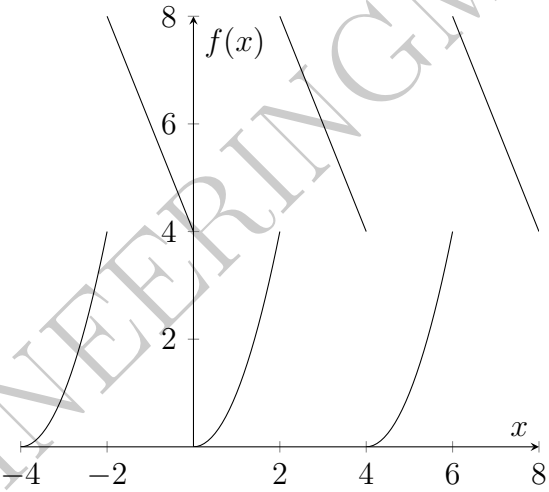
As I have already mentioned before the function $f(x)$ has a period 4. So the function $f(x)$ will repeat itself in the interval $[4, 8)$. Thus the function $f(x)$ will be

$$\begin{aligned} f(x) &= (x+4)^2 \quad 4 \le x < 6 \\ &= 4 - 2x \quad 6 \le x < 8. \end{aligned}$$

Next, I'll draw the graph of $f(x)$ in this interval.



Therefore I can say that the graph of $f(x)$, when x is in between -4 and 8, looks like



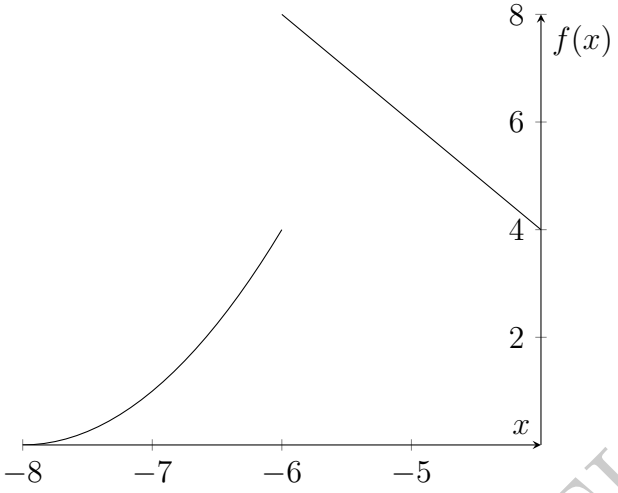
Now I'll draw the graph of $f(x)$ when x is in between -8 and -4.

Step 4

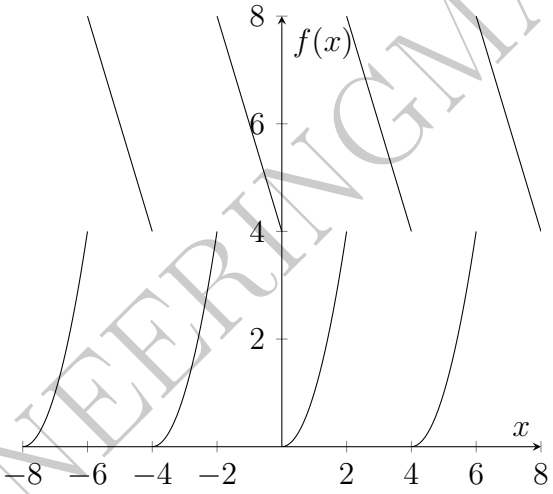
As I have mentioned before, the function $f(x)$ has a period 4. So the function $f(x)$ will repeat itself in the interval $[-8, -4)$.Thus the function $f(x)$ will be

$$\begin{aligned} f(x) &= (x+4)^2 \quad -8 \le x < -6 \\ &= 4 - 2x \quad -6 \le x < -4. \end{aligned}$$

Next, I'll draw the graph of $f(x)$ in this interval. So it will look like



Therefore I can say that the graph of $f(x)$, when x is in between -8 and 8, looks like



And this is the answer to this example.

Dear friends, this is the end of today's post on how to draw graphs in the Fourier series. Thank you very much for reading this. Please let me know how you feel about it. Soon I will be back again with a new post. Till then, bye, bye!!

References

[Stroud and Booth (2011)] , Advanced engineering mathematics, Industrial Press, Inc.; 5th Edition (March 8, 2011), Chapter: Fourier series 1, Further problems 17, p. 264, Q. No. 2(c) (Example 1), Q.No. 2(d) (Example 2), Q. No. 2(e) (Example 3).