Formulas in Fourier series

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1. Periodic function

$f(x+T) = f(x)$, T is the period of the function $f(x)$.

2. Fourier series of a function $f(x)$ with a period 2π

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx).$$

Here a_0, a_n, b_n are Fourier coefficients.

3. Fourier coefficients

$$\begin{aligned} a_0 &= \frac{1}{\pi}\int_{-pi}^{pi} f(x)\ dx, \\ a_n &= \frac{1}{\pi}\int_{-pi}^{pi} f(x)\cos nx\ dx, \\ b_n &= \frac{1}{\pi}\int_{-pi}^{pi} f(x)\sin nx\ dx, \end{aligned}$$

where $n = 1, 2, 3, \cdots$.

4. Dirichlet conditions

As per these conditions, the function $f(x)$ will have the above-mentioned Fourier series if

- the function $f(x)$ is defined, single-valued snd periodic with the period 2π,

- both $f(x)$ and $f^{'}(x)$ are peicewise continuous in the interval.

5. Formulas for specific integrals:

$$\int_{-\pi}^{\pi} dx = 2\pi.$$

$$\int_{-\pi}^{\pi} \cos nx \ dx = 0.$$

$$\int_{-\pi}^{\pi} \sin nx \ dx = 0.$$

$$\begin{aligned} \int_{-\pi}^{\pi} \cos ax \cos bx \ dx &= 1 \quad \text{if } a = b \\ &= 0 \quad \text{if } a \neq b. \end{aligned}$$

$$\begin{aligned} \int_{-\pi}^{\pi} \sin ax \sin bx \ dx &= \pi \quad \text{if } a = b \\ &= 0 \quad \text{if } a \neq b. \end{aligned}$$

$$\int_{-\pi}^{\pi} \cos mx \ \sin nx \ dx = 0.$$