Laplace transform of functions multiplied by variables

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Example 1

According to Stroud and Booth (2011)* "Determine the Laplace transform of the following function: $t \sin 3t$."

Solution

Laplace transform of $\sin 3t$ is

$$\begin{aligned}\mathcal{L}\{\sin 3t\} &= \frac{3}{(s)^2+(3)^2} \\ &= \frac{3}{s^2+9}.\end{aligned}$$

As per the formula of the Laplace transform of functions multiplied by variables, $\mathcal{L}\{t \sin 3t\}$ will be

$$\begin{aligned}\mathcal{L}\{t \sin 3t\} &= -\frac{d}{ds}\left\{\frac{3}{s^2+9}\right\} \\ &= -3\left\{-\frac{1}{(s^2+9)^2} \times 2s\right\} \\ &= \frac{6s}{(s^2+9)^2}.\end{aligned}$$

This is the answer to the given example.

Example 2

According to Stroud and Booth (2011)* "Determine the Laplace transform of the following function: $t^2 \cos t$."

Solution

I can rewrite the function $t^2 \cos t$ as $t(t \cos t)$. Laplace transform of the function $t \cos t$ will be $\mathcal{L}\{t \cos t\} = -\frac{d}{ds}\{F(s)\}$. So first I'll get the Laplace transform of the function $t \cos t$. Now from the standard formulas in Laplace transform, I can say that

$$\mathcal{L}\{\cos t\} = \frac{s}{(s)^2 + 1}.$$

Thus the Laplace transform of the function $t \cos t$ will be

$$\begin{aligned}
\mathcal{L}\{t \cos t\} &= -\frac{d}{ds}\left\{\frac{s}{s^2+1}\right\} \\
&= -\left[\frac{(s^2+1) - (s)2s}{(s^2+1)^2}\right] \\
&= \frac{s^2-1}{(s^2+1)^2}.
\end{aligned}$$

Next, I'll determine $\mathcal{L}\{t^2 \cos t\}$. Thus $\mathcal{L}\{t^2 \cos t\}$ will be

$$\begin{aligned}
\mathcal{L}\{t^2 \cos t\} &= -\frac{d}{ds}\left\{\frac{s^2-1}{(s^2+1)^2}\right\} \\
&= -\left[\frac{(s^2+1)^2(2s) - (s^2-1).2(s^2+1)(2s)}{(s^2+1)^4}\right] \\
&= -\left[\frac{2s(s^2+1) - 4s(s^2-1)}{(s^2+1)^3}\right] \\
&= \frac{2s^3-6s}{(s^2+1)^3}.
\end{aligned}$$

This is the answer to the given example.

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*Reference: K. A. Stroud and Dexter J. Booth (2011): Advanced engineering mathematics, Industrial Press, Inc.; 5th Edition (March 8, 2011), Chapter: Laplace transform 1, Test exercises 2, p. 90, Q. No. 1(e) (Example 1); Further problems 2, p. 90, Q. No. 1(e) (Example 2).