Example 1

According to Stroud and Booth (2011)* "Determine the Laplace transform of the following function: \( t \sin 3t \)."

Solution

Laplace transform of \( \sin 3t \) is

\[
\mathcal{L}\{\sin 3t\} = \frac{3}{(s^2 + (3)^2)} = \frac{3}{s^2 + 9}.
\]

As per the formula of the Laplace transform of functions multiplied by variables, \( \mathcal{L}\{t \sin 3t\} \) will be

\[
\mathcal{L}\{t \sin 3t\} = -\frac{d}{ds} \left\{ \frac{3}{s^2 + 9} \right\} = -3 \left\{ -\frac{1}{(s^2 + 9)^2} \times 2s \right\} = \frac{6s}{(s^2 + 9)^2}.
\]

This is the answer to the given example.
Example 2

According to Stroud and Booth (2011)* ”Determine the Laplace transform of the following function: $t^2 \cos t$.”

Solution

I can rewrite the function $t^2 \cos t$ as $t(t \cos t)$. Laplace transform of the function $t \cos t$ will be $\mathcal{L}\{t \cos t\} = -\frac{d}{ds}\{F(s)\}$. So first I’ll get the Laplace transform of the function $t \cos t$. Now from the standard formulas in Laplace transform, I can say that

$$\mathcal{L}\{\cos t\} = \frac{s}{(s^2 + 1)}.$$  

Thus the Laplace transform of the function $t \cos t$ will be

$$\mathcal{L}\{t \cos t\} = -\frac{d}{ds}\left\{\frac{s}{s^2 + 1}\right\}$$

$$= -\left[\frac{(s^2 + 1) - (s)2s}{(s^2 + 1)^2}\right]$$

$$= \frac{s^2 - 1}{(s^2 + 1)^2}.$$  

Next, I’ll determine $\mathcal{L}\{t^2 \cos t\}$. Thus $\mathcal{L}\{t^2 \cos t\}$ will be

$$\mathcal{L}\{t^2 \cos t\} = -\frac{d}{ds}\left\{\frac{s^2 - 1}{(s^2 + 1)^2}\right\}$$

$$= -\left[\frac{(s^2 + 1)^2(2s) - (s^2 - 1)2(s^2 + 1)(2s)}{(s^2 + 1)^4}\right]$$

$$= -\left[\frac{2s(s^2 + 1) - 4s(s^2 - 1)}{(s^2 + 1)^3}\right]$$

$$= \frac{2s^3 - 6s}{(s^2 + 1)^3}.$$  

This is the answer to the given example.