# Second-order homogeneous ODE with real, different roots

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## Example 1

According to Kreyszig (2005)*Find the general solution of the following: $y'' - 6y' - 7y = 0$.

Solution

Here the given second-order differential equation is

$$y'' - 6y' - 7y = 0.$$

Therefore the auxiliary equation (characteristic equation) will be

$$m^2 - 6m - 7 = 0.$$

Now I'll solve this equation to get the values of $m$. So I can also rewrite this equation as

$$\begin{aligned} m^2 - 6m - 7 &= 0 \\ m^2 + m - 7m - 7 &= 0. \end{aligned}$$

This means

$$(m + 1)(m - 7) = 0.$$

So I can say $m = -1$ or $m = 7$. That shows the given differential equation has two real different roots. Therefore the general solution of the given differential equation is

$$y = C_1 e^{-x} + C_2 e^{7x}.$$

This is the solution to the given equation. Next, I'll show you another example.

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## Example 2

According to Kreyszig (2005)*Find the general solution of the following: $10y'' - 7y' + 1.2y = 0$.

Solution

Here the given second-order differential equation is

$$10y'' - 7y' + 1.2y = 0.$$

Therefore the auxiliary equation (characteristic equation) will be

$$10m^2 - 7m + 1.2 = 0.$$

Now I'll solve this equation to get the values of $m$. So I can also rewrite this equation as

$$100m^2 - 70m + 12 = 0.$$

Now I can divide this equation throuth out by 2 to get

$$50m^2 - 35m + 6 = 0.$$

Next, I'll factorise it to get

$$\begin{aligned} 50m^2 - 35m + 6 &= 0 \\ 50m^2 - 20m - 15m + 6 &= 0 \\ 10m(5m - 2) - 3(5m - 2) &= 0. \end{aligned}$$

This means

$$(5m - 2)(10m - 3) = 0.$$

So I can say $m = \dfrac{2}{5}$ or $m = \dfrac{3}{10}$. That shows the given differential equation has two real different roots. Therefore the general solution of the given differential equation is

$$y = C_1 e^{2/5x} + C_2 e^{3/10x}.$$

Hence this is the solution to the given equation. Now I'll give another example.

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## Example 3

According to Kreyszig (2005)* Find the general solution of the following: $100y'' + 20y' - 99y = 0$.

Solution

Here the given second-order differential equation is

$$100y'' + 20y' - 99y = 0.$$

Therefore the auxiliary equation (characteristic equation) will be

$$100m^2 + 20m - 99 = 0.$$

Now I'll solve this equation to get the values of $m$.So I can also rewrite this equation as

$$\begin{aligned} 100m^2 + 20m - 99 &= 0 \\ 100m^2 + 110m - 90m - 99 &= 0 \\ 10m(10m + 11) - 9(10m + 11) &= 0. \end{aligned}$$

This means

$$(10m + 11)(10m - 9) = 0.$$

So I can say $m = -\dfrac{11}{10}$ or $m = \dfrac{9}{10}$. That shows the given differential equation has two real different roots. Therefore the general solution of the given differential equation is

$$y = C_1 e^{(9/10)x} + C_2 e^{-(11/10)x}.$$

Hence I can conclude that this is the solution to the given equation.

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Kreyszig, E. (2005)*: Advanced Engineering Mathematics: International Edition, John Wiley and Sons, 9th Edition, 29th December 2005, Chapter 2, Second-order linear ODEs, p. 59, Problem set 2.2, Q. 1(Example 1), Q. 2 (Example 2), Q. 3 (Example 5).