Second-order homogeneous ODE with real, equal roots

Dr. Aspriha Peters
Engineering math blog
https://www.engineeringmathgeek.com/
engineeringmathgeek@gmail.com

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Example 1

According to Kreyszig (2005)*Find the general solution of the following: $y'' + 4\pi y' + 4\pi^2 y = 0$.

<u>Solution</u>

Here the given second-order differential equation is

$$y'' + 4\pi y' + 4\pi^2 y = 0.$$

Therefore the auxiliary equation (characteristic equation) will be

$$m^2 + 4\pi m + 4\pi^2 = 0.$$

Now I'll solve this equation to get the values of m.

$$\begin{aligned} m^2 + 4\pi m + 4\pi^2 &= 0 \\ (m + 2\pi)^2 &= 0. \end{aligned}$$

Therefore I can say $m = -2\pi, -2\pi$. This shows that the given differential equation has two real equal roots. Therefore the general solution of the given differential equation is

$$y = (C_1 + C_2 x)e^{-2\pi x}.$$

Hence this is the solution to the given equation. Next, I'll show you another example.

Example 2

According to Kreyszig (2005)*Solve the initial value problem: $y'' + 2y' + y = 0, y(0) = 4, y'(0) = -6$.

Solution

Here the given second-order differential equation is

$$y'' + 2y' + y = 0.$$

Therefore the auxiliary equation (characteristic equation) will be

$$m^2 + 2m + 1 = 0.$$

Now I'll solve this equation to get the values of m.

$$\begin{aligned} m^2 + 2m + 1 &= 0 \\ (m+1)^2 &= 0. \end{aligned}$$

So I can say $m = -1, -1$. That shows that the given differential equation has two real equal roots. Therefore the general solution of the given differential equation is

$$y = (C_1 + C_2 x)e^{-x}. \quad (1)$$

Also it is given that $y(0) = 4, y'(0) = -6$. This means at $x = 0$, $y = 4$ and $y' = -6$. So, first I'll put $x = 0$ and $y = 4$ in equation (1). Therefore it will be

$$4 = (C_1 + C_2.0)e^{-0}.$$

That means

$$4 = (C_1 + 0).1$$

This gives $C_1 = 4$. Now I'll differentiate equation (1) with respect to x. This gives

$$y' = (C_2)e^{-x} - (C_1 + C_2 x)e^{-x}. \quad (2)$$

Next I'll put $x = 0, y' = -6$ and $C_1 = 4$ in equation (2). This means

$$-6 = (C_2)e^{-0} - (4 + C_2.0)e^{-0}.$$

That means

$$-6 = (C_2) - (4)1.$$

This gives $C_2 = -2$. Thus the solution of the differential equation is $y = (4-2x)e^{-x}$. Hence this is the solution to the given equation.

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*Kreyszig, E. (2005): Advanced Engineering Mathematics: International Edition, John Wiley & Sons, 9th Edition, 29th December 2005, Chapter 2, Second-order linear ODEs, p. 59, Problem set 2.2, Q. 4(Example 1), Q. 22 (Example 2).