Partial fractions of lower degree numerators

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Now I will talk about partial fractions of lower degree numerators.

Partial fractions of lower degree numerators

Let me solve some examples.

Example 1

Write the following expression into a partial fraction form:

$\dfrac{5x-18}{x^2-8x+12}$.

Solution

First of all, I'll factorize $(x^2-8x+12)$. So I can write $(x^2-8x+12)$ as

$$x^2-8x+12=(x-2)(x-6).$$

Now I can rewrite the expression $\left(\dfrac{5x-18}{x^2-8x+12}\right)$ as

$$\frac{5x-18}{x^2-8x+12}=\frac{5x-18}{(x-2)(x-6)}.$$

Let me assume

$$\frac{5x-18}{(x-2)(x-6)} = \frac{A}{x-2} + \frac{B}{x-6}$$

where A, B are constants.

Now

$$\frac{A}{x-2} + \frac{B}{x-6} = \frac{A(x-6)+B(x-2)}{(x-2)(x-6)}$$

Therefore I can say,

$$\begin{aligned}\frac{5x-18}{(x-2)(x-6)} &= \frac{A(x-6)+B(x-2)}{(x-2)(x-6)} \\ &= \frac{x(A+B)-(6A+2B)}{(x-2)(x-6)}.\end{aligned}$$

Now I compare numerators of both sides.

$$5x-18 \equiv x(A+B)-(6A+2B).$$

I get two equations with two unknowns A and B like

$$A+B=5 \tag{1}$$

and

$$-(6A+2B)=-18. \tag{2}$$

Now I'll solve equations (1) and (2) to get the values of A and B. So this gives $A=2, B=3$. Thus the partial fraction of $\frac{5x-18}{x^2-8x+12}$ is $\frac{2}{x-2}+\frac{3}{x-6}$. This is the answer to this example.

Now I'll give some other examples.

Disclaimer: None of these examples is mine. I have chosen these from some book or books. I have also given the due reference at the end of the post.

Example 2

According to Stroud and Booth (2013)*, "Express each of the following in partial fractions: $\frac{3x-9}{x^2-3x-18}$."

Solution

First of all, I have to factorize the denominator $(x^2 - 3x - 18)$ as

$$x^2 - 3x - 18 = (x+3)(x-6).$$

Therefore I can rewrite the expression $\left(\dfrac{3x-9}{x^2-3x-18}\right)$ as

$$\frac{3x-9}{x^2-3x-18} = \frac{3x-9}{(x+3)(x-6)}.$$

Let me assume

$$\frac{3x-9}{(x+3)(x-6)} = \frac{A}{x+3} + \frac{B}{x-6}$$

where A, B are constants. Now I'll simplify $\dfrac{A}{x+3} + \dfrac{B}{x-6}$ as

$$\frac{A}{x+3} + \frac{B}{x-6} = \frac{x(A+B) + (-6A+3B)}{(x+3)(x-6)}.$$

Therefore I can say that

$$\frac{3x-9}{(x+3)(x-6)} = \frac{x(A+B) + (-6A+3B)}{(x+3)(x-6)}.$$

Now I'll compare the numerators of both sides. That is

$$3x - 9 \equiv x(A+B) + (-6A+3B).$$

From here I get two equations with two unknowns A and B. So these are

$$A + B = 3 \tag{3}$$

and

$$-6A + 3B = -9.$$

Now I can also rewrite the second equation as

$$-2A + B = -3. \tag{4}$$

Next, I'll solve equations (3) and (4) to get the values of A and B. So this gives $A = 2, B = 1$. Thus the partial fraction of $\dfrac{3x-9}{x^2-3x-18}$ is $\dfrac{2}{x+3} + \dfrac{1}{x-6}$. Hence I can conclude that this is the answer to the given example. Now I'll give another example.

Example 3

According to Stroud and Booth (2013)*, "Express each of the following in partial fractions: $\dfrac{7x-7}{6x^2+11x+3}$."

Solution

First of all I'll factorize $(6x^2+11x+3)$ as

$$6x^2+11x+3=(3x+1)(2x+3).$$

Now I can rewrite the expression $\dfrac{7x-7}{6x^2+11x+3}$ as

$$\frac{7x-7}{6x^2+11x+3}=\frac{7x-7}{(3x+1)(2x+3)}.$$

Let me assume

$$\frac{7x-7}{(3x+1)(2x+3)}=\frac{A}{3x+1}+\frac{B}{2x+3}$$

where A, B are constants. Now I'll simplify $\dfrac{A}{3x+1}+\dfrac{B}{2x+3}$ as

$$\frac{A}{3x+1}+\frac{B}{2x+3}=\frac{x(2A+3B)+(3A+B)}{(3x+1)(2x+3}.$$

Now I'll compare the numerators of both sides. That is

$$7x-7\equiv x(2A+3B)+(3A+B).$$

From here I get two equations with two unknowns A and B. So these are

$$2A+3B=7 \tag{5}$$

and

$$3A+B=-7. \tag{6}$$

Next, I'll solve equations (5) and (6) to get the values of A and B. So this gives $A=-4, B=5$. Thus the partial fraction of $\dfrac{7x-7}{6x^2+11x+3}$ is $-\dfrac{4}{3x+1}+\dfrac{5}{2x+3}$. Hence I can conclude that this is the answer to the given example.

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*Reference: K. A. Stroud and Dexter J. Booth (2013): Engineering mathematics, Industrial Press, Inc.; 7th Edition (March 8, 2013), Chapter: Partial fractions, Further problems F.8, p. 240, Q. No. 4 (Example 2), Q. No. 8 (Example 3).