Heaviside unit step function

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Example 1

According to Stroud and Booth (2011)* "Express in terms of the Heaviside unit step function

$$\begin{aligned} f(t) &= t^2 \quad 0 \leq t < 3 \\ &= 5t \quad 3 \leq t. \text{ "} \end{aligned}$$

Solution

The given function is:

$$\begin{aligned} f(t) &= t^2 \quad 0 \leq t < 3 \\ &= 5t \quad 3 \leq t. \end{aligned}$$

The function $f(t)$ has a break at $t = 3$. I'll write this function in such a way so that the first part of the function $f(t) = t^2$ is switched off at $t = 3$. And, the second part of the function $f(t) = 5t$ is switched on at $t = 3$. Therefore, in the unit step form, the function will be

$$f(t) = t^2u(t) - t^2u(t-3) + u(t-3)(5t).$$

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Next, I'll simplify the function in unit step form to get

$$f(t) = t^2u(t) - (t^2 - 5t)u(t-3).$$

Hence I can conclude that this is the answer to the given example.

Now I'll give another example.

Example 2

According to Stroud and Booth (2011)*, "Express in terms of the Heaviside unit step function

$$\begin{aligned} f(t) &= \cos t \quad 0 \leq t < \pi \\ &= \cos 2t \quad \pi \leq t < 2\pi \\ &= \cos 3t \quad 2\pi \leq t. \text{ "} \end{aligned}$$

Solution

The given function is:

$$\begin{aligned} f(t) &= \cos t \quad 0 \leq t < \pi \\ &= \cos 2t \quad \pi \leq t < 2\pi \\ &= \cos 3t \quad 2\pi \leq t. \end{aligned}$$

As I can see, here the function $f(t)$ has two breaks - one is at $t = \pi$ and the other one is at $t = 2\pi$. I'll write this function in such a way so that the first part of the function $f(t) = \cos t$ is switched off at $t = \pi$. Also, the second part of the function $f(t) = \cos 2t$ is switched on at $t = \pi$. Next, the function $f(t) = \cos 2t$ is switched off at $t = 2\pi$. Finally, the function $f(t) = \cos 3t$ is switched on at $t = 2\pi$. Therefore, in the unit step form, the function will be

$$f(t) = \cos tu(t) - \cos tu(t-\pi) + \cos 2tu(t-\pi) - \cos 2tu(t-2\pi) + \cos 3tu(t-2\pi).$$

Now I'll simplify it to get

$$f(t) = \cos tu(t) + (\cos 2t - \cos t)u(t-\pi) + (\cos 3t - \cos 2t)u(t-2\pi).$$

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Hence I can conclude that this is the answer to the given example.

Now I'll give another example.

Example 3

According to Stroud and Booth (2011)*, "A function $f(t)$ is defined by

$$\begin{aligned} f(t) &= t^2 \quad 0 \leq t < 2 \\ &= 4 \quad 2 \leq t < 5 \\ &= 0 \quad 5 \leq t. \end{aligned}$$

Determine the function in terms of the unit step function."

Solution

The given function is:

$$\begin{aligned} f(t) &= t^2 \quad 0 \leq t < 2 \\ &= 4 \quad 2 \leq t < 5 \\ &= 0 \quad 5 \leq t. \end{aligned}$$

As I can see, here the function $f(t)$ has two breaks - one is at $t = 2$ and the other one is at $t = 5$. So I'll write this function in such a way so that the first part of the function $f(t) = t^2$ is switched off at $t = 2$. Also, the second part of the function $f(t) = 4$ is switched on at $t = 2$. Next, the function $f(t) = 4$ is switched off at $t = 5$. Finally, the function $f(t) = 0$ is switched on at $t = 5$. Therefore, in the unit step form, the function will be

$$f(t) = t^2u(t) - t^2u(t-2) + 4u(t-2) - u(t-5).4 + u(t-5).0.$$

Now I'll simplify it to get

$$f(t) = t^2u(t) - t^2u(t-2) + 4u(t-2) - 4u(t-5).$$

Hence I can conclude that this is the answer to the given example.

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*Reference: K. A. Stroud and Dexter J. Booth (2011): Advanced engineering mathematics, Industrial Press, Inc.; 5th Edition (March 8, 2011), Chapter: Laplace transform 2, Further problems 3, p. 121, Q. No. 4(a) (Example 1), Q. No. 4(b) (Example 2), Q. No. 6(a) (Example 3).

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