Example 1

According to Stroud and Booth (2011)* Determine whether the three vectors $A = 2i + 3j - 5k; B = i - 2j + 2k; C = 3i + j + 3k$ are coplanar.

Solution

Now here the given three vectors are:

\[
A = 2i + 3j - 5k, \\
B = i - 2j + 2k \\
C = 3i + j + 3k.
\]

Any three vectors will be coplanar if the scalar triple product of them is zero. Thus my next job is to find out the scalar triple product of these three vectors. Next, I will find out the scalar triple product $A \cdot (B \times C)$.

\[
\begin{vmatrix}
2 & 3 & 1 \\
1 & -2 & 2 \\
3 & 1 & 3
\end{vmatrix}
\]

Therefore it will be $A \cdot (B \times C) = \begin{vmatrix}
2 & 3 & 1 \\
1 & -2 & 2 \\
3 & 1 & 3
\end{vmatrix}$. 


Now I’ll evaluate the determinant on the right-hand side to get the value of $A \cdot (B \times C)$. Thus it will be

\[
A \cdot (B \times C) = (2)[(-2)(3) - (2)(1)] - (3)[(1)(3) - (2)(3)]
+ (1)[(1)(1) - (3)(-2)]
= (2)[-6 - 2] - (3)[3 - 6] + (1)[1 + 6]
= -16 + 9 + 7
= 0.
\]

So I can see that the scalar triple product $A \cdot (B \times C)$ equals to zero. Hence I can conclude that these three vectors are coplanar. And this is the answer to the given example.

Now I will move to the next one.

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**Example 2**

According to Stroud and Booth (2011)* “Determine the value of $p$ such that the three vectors $A, B, C$ are coplanar when $A = 2i + j + 4k; B = 3i + 2j + pk; C = i + 4j + 2k.$”

**Solution**

Now in this example, the given vectors are

$A = 2i + j + 4k,$

$B = 3i + 2j + pk$

and

$C = i + 4j + 2k.$

Next, I’ll determine the value of $p$ so that these three vectors will be coplanar. As I have already mentioned earlier, for coplanar vectors, the scalar triple product
will be zero. Thus the scalar triple product $A \cdot (B \times C)$ will be $A \cdot (B \times C) = \begin{vmatrix} 2 & 1 & 4 \\ 3 & 2 & p \\ 1 & 4 & 2 \end{vmatrix}$.

Now I'll evaluate the determinant as

\[
A \cdot (B \times C) = (2)[(2)(2) - (p)(4)] - (1)[(2)(3) - (p)(1)] \\
+ (4)[(3)(4) - (2)(1)] \\
= (2)[4 - 4p] - (1)[6 - p] + (4)[12 - 2] \\
= 8 - 8p - 6 + p + 40 \\
= -7p + 42.
\]

Now here I already know that these three vectors $A$, $B$, and $C$ are coplanar. Therefore the scalar triple product of these vectors $A \cdot (B \times C)$ equals to zero.

This means $(-7p + 42) = 0$. Next, I'll solve it to get the value of $p$. Thus I get

\[
-7p + 42 = 0 \\
p = 6.
\]

Thus I can conclude that for $p = 6$, the three vectors $A$, $B$, $C$ are coplanar when $A = 2i + j + 4k; B = 3i + 2j + pk; C = i + 4j + 2k$.

So here ends my second as well as the last example.