Polar form of a complex number

Dr. Aspriha Peters
Engineering math blog

https://www.engineeringmathgeek.com/
engineeringmathgeek@gmail.com

October 2, 2019

Example 1

According to Kreyszig (2005)* "Represent in polar form. \( \frac{1}{2} + \frac{1}{4} \pi i. \)"

Solution

First of all, I'll give the complex number a name, say, \( z \). So it will be \( z = \frac{1}{2} + \frac{1}{4} \pi i \).

Now I'll get the modulus of \( z \). So it will be

\[
|z| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{4} \pi \right)^2} = \frac{\sqrt{4 + \pi^2}}{4}.
\]

Next, I'll get the argument of \( z \). So let me choose the argument of \( z \) is \( \theta \). Hence \( \theta \) will be

\[
\theta = \tan^{-1} \frac{\pi/4}{1/2} = \tan^{-1} \frac{\pi}{2}.
\]
Thus the polar form of $z$ will be

$$z = \frac{\sqrt{4 + \pi^2}}{4} \left[ \cos(\tan^{-1}\frac{\pi}{2}) + i \sin(\tan^{-1}\frac{\pi}{2}) \right].$$

Hence I can conclude that this is the answer to the given example.

---

**Example 2**

According to Kreyszig (2005)* "Represent in polar form. $-5$.”

**Solution**

First of all, I’ll give the complex number a name, say, $z$. So it will be $z = -5 + 0i$.

Now I’ll get the modulus of $z$. So it will be

$$|z| = \sqrt{(-5)^2} = 5.$$  

Next, I’ll get the argument of $z$. So I’ll choose the argument of $z$ is $\theta$. Hence $\theta$ will be

$$\theta = \tan^{-1}\frac{0}{-5} = \tan^{-1}0 = \pi.$$  

Thus the polar form of $z$ will be

$$z = 5 \left[ \cos \pi + i \sin \pi \right].$$

Hence I can conclude that this is the answer to the given example.

---

**Example 3**

According to Kreyszig (2005)* "Represent in polar form. $\frac{1+i}{1-i}$.”
Solution

Let’s say $z = \frac{1+i}{1-i}$. Now I’ll remove the imaginary part from the denominator. So, I’ll multiply both the top and bottom of $z$ with $(1 + i)$.

\[
z = \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{(1+i)^2}{(1-i^2)} = i.
\]

Next, I’ll get the modulus of $z$. So it will be

\[
|z| = \sqrt{0^2 + 1^2} = 1.
\]

Then I’ll get the argument of $z$. So let me choose the argument of $z$ is $\theta$. Hence $\theta$ will be

\[
\theta = \tan^{-1} \frac{1}{0} = \tan^{-1} \infty = \frac{\pi}{2}.
\]

Thus the polar form of $z$ will be

\[
z = 1 \left[ \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right].
\]

Hence I can conclude that this is the answer to the given example.

Example 4

According to Kreyszig (2005)* "Represent in polar form. $\frac{2+3i}{5+4i}$."
Solution

Let’s say \( z = \frac{2 + 3i}{5 + 4i} \). First I’ll remove the imaginary part from the denominator of \( z \). So I’ll multiply both the top and bottom of \( z \) with \((5 - 4i)\). Thus it will be

\[
z = \frac{(2 + 3i)(5 - 4i)}{(5 + 4i)(5 - 4i)} = \frac{22}{41} + i \frac{7}{41}.
\]

Next, I’ll get the modulus of \( z \). So it will be

\[
|z| = \sqrt{\left(\frac{22}{41}\right)^2 + \left(\frac{7}{41}\right)^2} = \sqrt{\frac{13}{41}}.
\]

Then I’ll get the argument of \( z \). So let me choose the argument of \( z \) is \( \theta \). Hence \( \theta \) will be

\[
\theta = \tan^{-1} \frac{7/41}{22/41} = \tan^{-1} \frac{7}{22}.
\]

Thus the polar form of \( z \) will be

\[
z = \sqrt{\frac{13}{41}} \left[ \cos(\tan^{-1} \frac{7}{22}) + i \sin(\tan^{-1} \frac{7}{22}) \right] .
\]

Hence I can conclude that this is the answer to the given example.